京都大学若手人材海外派遣事業 ジョン万プログラム
研究者派遣プログラム

成果報告書

<div align="right">提出日：平成 ２９年 ９月１９日</div>

［基本情報］

〇申請者

採 択 年 度：平成２７年度

部 局 名 等：理学研究科数学専攻

職　　　　名：助教

氏　　　　名：塚本　真輝

研 究 課 題 名：力学系の平均次元

〇渡航先

国　　　　名：イスラエル

研 究 機 関 名：ヘブライ大学　アインシュタイン研究所

研 究 室 名 等：　［研究室名］エロン　リンデンシュトラウス

　　　　　　　　　［職名等・氏名］教授　エロン　リンデンシュトラウス（Elon Lindenstrauss）

渡 航 期 間：平成２８年３月２６日〜平成２９年３月８日（３４８日）

〇渡航期間中の出張

特になし.

［成果］
〇プロジェクトの成果及び今後の展開
・研究概要

研究上は非常に有意義な滞在であった．まず受け入れ先のエロン・リンデンシュトラウス氏とは頻繁に議論を重ね，その成果として「平均次元理論とレート歪み理論との間の変分原理」という成果を得た．これは力学系理論で非常に有名な定理である「変分原理」の本質的に新しいヴァージョンであり，今後のさらなる展開が期待される．論文は現在，学術雑誌に投稿中である．この他にも，リンデンシュトラウス氏とは平均次元に関わる多くの問題について頻繁に議論し，アイデアを交換した．上述の変分原理の研究を除いて，まだ論文にまとまるまでには至っていないが，将来，必ずや大きな結果につながるだろうと確信している．
次に，ポーランド在住のヨナタン・グットマン氏が一時，ヘブライ大学に滞在したのを良い機会として，氏と議論を重ねた．その成果として，「信号処理の手法の，マルチパラメータ力学系の埋め込み問題への応用」と「ベブートフ・角谷の定理の新ヴァージョン」という二つの結果を得た．前者はエロン・リンデンシュトラウス氏，ヨナタン・グットマン氏，私の３人で以前に書いた論文の結果を最善の形に拡張するものであり，後者はベブートフ・角谷による５０年ほど前の定理をより良い形に改善する成果である．後者の論文は既に学術誌に投稿しており，前者の論文は初稿が出来上がって，現在グットマン氏が細部を確認中である．
またフランス・フーリエ研究所のエルブ・ガウジエ氏とも議論を重ね，擬等角写像という古典的な題材で論文を書いた．学術誌に現在投稿中である．
このように，一年ほどの滞在で論文を４本執筆することができたのは，かなりの成果であると考えている．

・国際共同研究の立上げ・ネットワークの構築

上でも既に既述したように，国際共同研究によって，学問上の豊かな成果が得られている．今後もさらに発展させていきたい．また，共同研究・論文執筆には至らないまでも，ヘブライ大学の非常に優秀な研究者たちからはさまざまな刺激を受けることができた．今後の研究の展開に確実に生きてくるだろうと確信している．

・国際共著論文の投稿・発表等の状況、国際学会等での発表状況［予定を含む］

（１）国際共著論文：From rate distortion theory to metric mean dimension, Elon Lindenstrauss, Masaki Tsukamoto,学術誌に投稿中.
（２）国際共著論文：A Lipschitz refinement of the Bebutov-Kakutani theorem, Yonatan Gutman, Lei Jin, Masaki Tsukamoto, 学術誌に投稿中.
（３）国際共著論文：On the energy of quasiconformal mappings, Herve Gaussier, Masaki Tsukamoto, 学術誌に投稿中.
（４）国際共著論文：Application of signal analysis to the embedding problem of Z^k actions, Yonatan Gutman, Yixiao Qiao, Masaki Tsukamoto, 初稿が完成し，共著者が原稿チェック中.

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・在外研究経験によって習得した能力等

数学の場合，研究は少人数の議論によって進むのが通常であるため，今回のイスラエル滞在で研究室の運営方針等で新たに得たことなどは特にない．既に上の欄で記述したように，純粋に学問上の多くの進展があり，その成果は今後も間違いなく発展していくだろう．

　また，直接的にヘブライ大学での研究交流で得たことがらではないが，ヘブライ大学に隣接する科学博物館に非常に感銘を受けた．週末に家族で良く訪れたのだが，とても練りこまれた展示の数々に，子供と一緒に夢中になった．今後，日本でアウトリーチング活動をする際に必ず役立つ経験になったと思う．

・在外研究経験を活かした今後の展開

今回の国際共同研究で得られた成果は，どれもさらに発展可能なテーマであるので，研究を継続していきたい．特にエロン・リンデンシュトラウス氏と共同で継続中の研究は重要であり，今後も力を注いでいく予定である．

英文成果報告書

○申請者情報

部　　局　　名： Department of Mathematics, Faculty of science
職　　　　　名： Assistant Professor
氏　　　　　名： Masaki Tsukamoto
研　究　課　題　名： Mean dimension of dynamical systems
渡　航　期　間：From 26th March 2016 to 8th March 2017.

○渡航先情報

国　　　　　名： Israel
研　究　機　関　名： Hebrew University, Einstein Institute
研　究　室　名　等： Elon Lindenstrauss
受入研究者名： Elon Lindenstrauss

○渡航報告

　The Einstein Institute of Mathematics in Hebrew Univeristy is a quite unique and wonderful environment for my study of mathematics. My main research theme is a study of mean dimension, which is an averaged number of parameters of dynamical systems. My host researcher in the Einstein Institute is Professor Elon Lindnestrauss. He is one of the pioneers of mean dimension theory and the world authority of dynamical systems theory (he won the Fields medal by his works in the field of dynamical systems). I cannot imagine a reaech place better than the Einstein Intitute for my study of mean dimension theory. Professor Elon Lindenstrauss and I have already jointly worked on mean dimension for more than 5 years. This stay gave me an execelent opportunity to enlarge our collaborations.

　The Einstein institute has a long and very nice tradition of dynamical systems theory, originally directed by Professor Hille Furstenberg (the Wolf prize winner) and Professor Benjamin Weiss for many years (probably more than half a century). Now their former students become world leading mathematicians (for examples, Elon Lindenstrauss, Alex Lubotzki, Yuver Peres, Mike Hochman, Tamar Ziegler....; this is a really surpring list of high level mathematicains) and many of them are professors of the Einstein Institute. So the study of dynamical systems is very active there. For example, there are usually three seminars on dynamical systems in one week. (This is quite high rate compared to the world standard.) This is an idealistic environment for me. I absorbed plenty of hot information every week, and conversations with the above mentioned high level mathematicians were always impressive.

　Another (a bit less important but quite nice) attractive point of the institute is its library. It has a unique style. First, it displays books according to the subjects (e.g. dynamical systems, probabilty, algebraic geometry, complex analysis, etc.). I love this style,which provides me an opportunity to meet unexpected nice books. Second, it is the library for both the mathematics department and the computer science department. So we can find not only math books but also computer science books. This is quite helpful for my research because mean dimension theory (and many other branches of

ergodic theory and dynamical systems) has deep connections with computer science. Indeed (as I will write below) one of my chief achievements during this visit is to find the new relation between mean dimension theory and rate distortion theory (one of the important branches of information theory, originally introduced by Claud Shannon).

As I described above, the environment of the Einstein Institute is quite nice. Influenced by this, my research was developed rapidly. The most important achievment is to find a new variational principle in dynamical systems theory. Variational principle is a classical result in dynamics, which originally connectes topological entropy with measure theoretic (Kolmogorov—Sinai) entropy. Jointly with Professor Elon Lindenstrauss, I found a totally new kind of variational principle. We proved that the metric mean dimension (an important quatity in mean dimension theory) of dynamical systems can be expressed by the rate distortion functions of the associated stochastic processes. This discovery connectes the mean dimension theory with the information theory more concretely and introduces probabilistic ideas in the context of mean dimension theory, which expand the scope of the theory substantially. The joint paper was already written up and submitted to a journal.

Besides the above mentioned work on new variational principle, Professor Elon Lindenstrauss and I also discussed other problems on foundational issues of mean dimension theory quite frequently. I am sure that it will result in another important development in a future.

I also developed other collaborations. Professor Yonatan Gutman (a math researcher in Poland) once visited the Einstein Institute. We discussed the embedding problem of multi-parameter dynamical systems (in technical terms, continuous actions of higher rank abelian groups on compact metric spaces). We found a very new approach to the problem, which deeply uses the ideas of communication theory. The first version of our joint paper was already written down. Now Proffessor Gutman is checking the manuscript. I hope that we will be able to submit the paper to a journal in a very near future.

I also worked on other problems: Jointly with Professor Yonatan Gutman, I proved a new version of Bebutov—Kakutani's theorem in dynamical syetems theory. Original Bebutov—Kakutani's theorem is a classical nice theorem (proved about 50 years ago) but it has one drawback. We found its more satisfactory (without the same drawback) refinment. The joint paper was already submitted to a journal.

Moreover I wrote a paper on quasi-conformal mappings (a classical theme in complex analysis) and pseudo-holomorphic curves jointly with Professor Herve Gaussier (a researcher in France). We found a new intersting connection of the subject with fractal geonetry. The paper was already submitted to a journal.

As a whole, I wrote four papers during the stay in Israel. I am sure that the nice environment of the Einstein Institute helped this high productivity.

Professor Elon Lindenstrauss gave me an opportunity to give a mini-course on mean dimension theory. It was a great pleasure to give a course in front of world-leading mathematicians. I think my course was succesful. I explained the solution to a problem posed by professor Elon Lindenstrauss in 1999, which is probably one of the most beatiful developments of mean dimension theory in the last

decade. (This is a joint work with Professor Yonatan Gutman.) One audience kindly told me that our result is quite nice.

Hebrew University is located in Jerusalem, which is certainly one of the most historical places in the world. The history of the city is very long and complicated. Staying in this historical city was an influencial experience for me. Many delicate (socail, political, religious) issues remain and TV news around the world frequently report several affairs on Jerusalem. It is true that many troublesome issues sometimes happen in Jerusalem. But I love this city and people there. The city has many interesting traditions. The best one (for me) is Yom Kippur. It is the most holy day in year. Surprisingly, no one drive a car on Yom Kippur. So people can walk on every road. It is a realy impressive experience to walk on big roads which have many (possibly 10) traffic lanes. There also exist many other religious activities all over the year. They are kind of experiences that we never see in Japan (or, probably, in most Western countries).

My family often went to the science museum in Jerusalem close to the Hebrew University in the weekend. This museum is very nice. It is not so big but its exhibitions are highly elaborated. Childrean (and also their parents) can learn science very pleasently though the exhibitions. It is a shameful fact that Japan does not have such a nice science museum although Japan is a much larger country than Israel. The experience of this museum will be helpful in the future when I try to do some out-reaching activities.

As a conlusion, my stay in the Einstein Institute was very successful from the view point of mathematics.